Practice Exam 1

CS 2300

1. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

<u>Contrapositive</u>:	"If you don't sleep late, then it is not Saturday."
<u>Converse</u>:	"If you sleep late, then it is Saturday."
<u>Inverse</u>:	"If it is not Saturday, then you don't sleep late."

2. Translate the given statement into propositional logic using the propositions provided: On certain highways in the Washington, DC metro area you are allowed to travel on high occupancy lanes during rush hour only if there are at least three passengers in the vehicle. Express your answer in terms of r:"You are traveling during rush hour." t:"You are riding in a car with at least three passengers." and h:"You can travel on a high occupancy lane."

$$(r \wedge h) \rightarrow t$$

3. Is the proposition $(\neg p \vee \neg q) \wedge (p \rightarrow q)$ satisfiable?

Yes. Try all 4 possibilities. It is satisfiable by (p=F, q=T) and (p=F, q=F).

In 4–6, P(m,n) means "m ≤ n", where the universe of discourse for m and n is the set of nonnegative integers. What is the truth value of each statement?

4. $\forall n P(0,n)$

"Zero is less than or equal to all nonnegative integers." **True**.

5. $\exists n \forall m\, P(m,n)$

"There is a nonnegative integer greater than or equal to all nonnegative integers." **False**.

6. $\forall m \exists n\, P(m,n)$

"For each nonnegative integer, there is a nonnegative integer greater than or equal to it." **True**.

In 7–9, suppose the variable x represents students and y represents courses, and: M(y): y is a math course F(x): x is a freshman B(x): x is a full-time student T (x, y): x is taking y. Write the statement in good English without using variables in your answers.

7. $\exists x \forall y\ T(x,y)$

"There is a student taking every course."

8. $\forall x \exists y\ T(x,y)$

"Every student is taking a course."

9. $\forall x \exists y\ [(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x,y))]$

"All full-time freshmen are taking a math course."

In 10–12, suppose the variable x represents people, and F(x): x is friendly T(x): x is tall A(x): x is angry. Write the statement using these predicates and any needed quantifiers.

10. Some people are not angry.

$\exists x \neg A(x)$ or $\neg \forall x A(x)$

11. All tall people are friendly.

$$\forall x\big(T(x) \rightarrow F(x)\big)$$

12. No friendly people are angry.

$$\forall x\big(F(x) \rightarrow \neg A(x)\big) \equiv \forall x\big(A(x) \rightarrow \neg F(x)\big) \equiv \neg \exists x\big(F(x) \wedge A(x)\big)$$

13. Determine whether the following argument is valid. If you are not in the tennis tournament, you will not meet Ed.If you aren't in the tennis tournament or if you aren't in the play, you won't meet Kelly. You meet Kelly or you don't meet Ed. It is false that you are in the tennis tournament and in the play. Therefore, you are in the tennis tournament.

Given:
1) $\neg t \rightarrow \neg e$
2) $(\neg t \vee \neg p) \rightarrow \neg k$
3) $k \vee \neg e$
4) $\neg t \vee \neg p$

Using 4 and 2, we conclude $\neg k$. By 3), then, we have $\neg e$. By 1, t can be either T or F, so we cannot conclude anything about t. **Not valid**.

14. Suppose you wish to prove a theorem of the form "if p then q".(a) If you give a direct proof, what do you assume and what do you prove?(b) If you give a proof by contraposition, what do you assume and what do you prove? (c) If you give a proof by contradiction, what do you assume and what do you prove?

a) Assume p, prove q.

b) Assume $\neg q$, prove $\neg p$.

c) Assume p and $\neg q$, then derive a contradiction (something that is impossible).

15. Consider the following theorem: If x is an odd integer, then x + 2 is odd. Give a direct proof of this theorem

$$x = 2n + 1 \Rightarrow x + 2 = 2n + 3 = 2(n + 1) + 1$$

The last quantity is an odd number.

16. Consider the following theorem: If x is an odd integer, then x + 2 is odd. Give a proof by contraposition of this theorem.

Assume $x+2 = 2n$. Then $x = 2n-2 = 2(n-1)$, an even number.

17. Prove: if m and n are even integers, then mn is a multiple of 4.

Let $m = 2p$, and $n = 2k$. Then $mn = 4pk$, a multiple of 4.